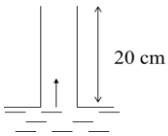
Solución Problema de difusión:

Un tubo hueco de 20 cm de longitud está inicialmente lleno con aire conteniendo 2% de etilalcohol (vapor). Al fondo del tubo está una piscina de alcohol el cual se evapora en el gas estancado de arriba



Ecuación diferencial

$$D\frac{\partial^2 C}{\partial x^2} = \frac{\partial C}{\partial t}$$

Condiciones de borde

t=0	c(x,0)=2
x=0	c(0,t)=0
x=20	c(20,t)=10

Coeficiente de difusión: D = 0.119

Utilizando 5 intervalos en la dirección x

ΔX = (b-a)/inter Δx = 4

Por lo que:

Para garantizar estabilidad se toma: λ= 0.50 ΔT = 67.2268908

Por el método explícito:

$$C_i^{\,j+1} = C_i^{\,j} + \lambda(C_{i+1}^{\,j} - 2C_i^{\,j} + C_{i-1}^{\,j})$$

Si λ= 0.50

$$C_i^{\,j+1} = \frac{1}{2}(C_{i+1}^{\,j} + C_{i-1}^{\,j})$$

Así se construye la siguiente tabla:

Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
0	0	2	2	2	2	10
67.2	0	1	2	2	6	10
134.4	0	1	1.5	4	6	10
201.6	0	0.75	2.5	3.75	7	10
268.8	0	1.25	2.25	4.75	6.875	10
336	0	1.125	3	4.5625	7.375	10
403.2	0	1.5	2.84375	5.1875	7.28125	10
470.4	0	1.421875	3.34375	5.0625	7.59375	10
537.6	0	1.671875	3.2421875	5.46875	7.53125	10
604.8	0	1.62109375	3.5703125	5.38671875	7.734375	10
672	0	1.78515625	3.50390625	5.65234375	7.69335938	10
739.2	0	1.75195313	3.71875	5.59863281	7.82617188	10
806.4	0	1.859375	3.67529297	5.77246094	7.79931641	10
873.6	0	1.83764648	3.81591797	5.73730469	7.88623047	10
940.8	0	1.90795898	3.78747559	5.85107422	7.86865234	10
1008	0	1.89373779	3.8795166	5.82806396	7.92553711	10

Ahora si: λ= 0.25 ΔT = 33.6134454

Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
0	0	2	2	2	2	10
33.6	0	1.5	2	2	4	10
67.2	0	1.25	1.875	2.5	5	10
100.8	0	1.09375	1.875	2.96875	5.625	10
134.4	0	1.015625	1.953125	3.359375	6.0546875	10
168	0	0.99609375	2.0703125	3.68164063	6.3671875	10
201.6	0	1.015625	2.20458984	3.95019531	6.60400391	10
235.2	0	1.05895996	2.34375	4.17724609	6.78955078	10
268.8	0	1.11541748	2.48092651	4.37194824	6.93908691	10
302.4	0	1.17794037	2.61230469	4.54097748	7.06253052	10
336	0	1.24204636	2.73588181	4.68919754	7.16650963	10
369.6	0	1.30499363	2.85075188	4.82019663	7.2555542	10
403.2	0	1.36518478	2.9566735	4.93667483	7.33282626	10
436.8	0	1.42176077	3.05380166	5.04071236	7.40058184	10
470.4	0	1.4743308	3.14251911	5.13395205	7.46046901	10
504	0	1.52279518	3.22333027	5.21772305	7.51372252	10
537.6	0	1.56723015	3.29679469	5.29312472	7.56129202	10
571.2	0	1.60781375	3.36348607	5.36108404	7.60392719	10
604.8	0	1.64477839	3.42396748	5.42239533	7.64223461	10
638.4	0	1.67838107	3.47877717	5.47774819	7.67671614	10
672	0	1.70888483	3.5284209	5.52774742	7.70779512	10
705.6	0	1.73654764	3.57336851	5.57292771	7.73583441	10
739.2	0	1.76161595	3.61405309	5.61376459	7.76114914	10
772.8	0	1.78432125	3.65087168	5.65068285	7.78401571	10
806.4	0	1.80487854	3.68418686	5.68406327	7.80467857	10
840	0	1.82348599	3.71432889	5.714248	7.8233551	10
873.6	0	1.84032522	3.74159794	5.741545	7.84023955	10
907.2	0	1.85556209	3.76626652	5.76623187	7.85550602	10
940.8	0	1.86934768	3.78858175	5.78855907	7.86931098	10
974.4	0	1.88181928	3.80876756	5.80875272	7.88179526	10
1008	0	1.89310153	3.82702678	5.82701706	7.89308581	10

Por el método implícito (Crank-Nicolson)

$$-\lambda C_{i-1}^{\,j+1} + 2(1+\lambda) C_i^{\,j+1} - \lambda C_{i+1}^{\,j+1} = \lambda C_{i-1}^{\,j} + 2(1-\lambda) C_i^{\,j} + \lambda C_{i+1}^{\,j}$$

Se toma por comodidad: $\lambda = 1.00$ Así: $\Delta T = 134.453782$

$$-C_{i-1}^{\,j+1} + 4C_i^{\,j+1} - C_{i+1}^{\,j+1} = C_i^{\,j} + C_{i+1}^{\,j}$$

Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
0	0	2	2	2	2	10

	C_1	C_2	C_3	C_4	b	sol
	4	-1	0	0	2	1.00478469
	-1	4	-1	0	4	2.01913876
	0	-1	4	-1	4	3.07177033
	0	0	-1	4	22	6.26794258

Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
134.4	0	1.00478469	2.01913876	3.07177033	6.26794258	10

	C_1	C_2	C_3	C_4	b	sol
	4	-1	0	0	2.01913876	1.10258465
	-1	4	-1	0	4.07655502	2.39119984
	0	-1	4	-1	8.28708134	4.38565967
	0	0	-1	4	23.0717703	6.8643575

Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
268.8	0	1.10258465	2.39119984	4.38565967	6.8643575	10

	C_1	C_2	C_3	C_4	b	sol
	4	-1	0	0	2.39119984	1.32841505
	-1	4	-1	0	5.48824432	2.92246035
	0	-1	4	-1	9.25555734	4.87318203
	0	0	-1	4	24.3856597	7.31471042

Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
403.2	0	1.32841505	2.92246035	4.87318203	7.31471042	10

	C_1	C_2	C_3	C_4	b	sol
	4	-1	0	0	2.92246035	1.54307943
	-1	4	-1	0	6.20159707	3.24985737
	0	-1	4	-1	10.2371708	5.25475297
	0	0	-1	4	24.873182	7.53198375

Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
537.6	0	1.54307943	3.24985737	5.25475297	7.53198375	10

	C_1	C_2	C_3	C_4	b	sol
	4	-1	0	0	3.24985737	1.68584505
	-1	4	-1	0	6.7978324	3.49352284
	0	-1	4	-1	10.7818411	5.49041392
	0	0	-1	4	25.254753	7.68629172

Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
672	0	1.68584505	3.49352284	5.49041392	7.68629172	10

	C_1	C_2	C_3	C_4	b	sol
	4	-1	0	0	3.49352284	1.78703749
	-1	4	-1	0	7.17625897	3.65462713
	0	-1	4	-1	11.1798146	5.65521205
	0	0	-1	4	25.4904139	7.78640649

Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
806.4	0	1.78703749	3.65462713	5.65521205	7.78640649	10

	C_1	C_2	C_3	C4	b	sol
	4	-1	0	0	3.65462713	1.85508234
	-1	4	-1	0	7.44224954	3.76570223
	0	-1	4	-1	11.4410336	5.76547703
	0	0	-1	4	25.655212	7.85517227

Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
940.8	0	1.85508234	3.76570223	5.76547703	7.85517227	10

	C_1	C_2	C_3	C_4	b	sol
	4	-1	0	0	3.76570223	1.90161096
	-1	4	-1	0	7.62055937	3.84074161
	0	-1	4	-1	11.6208745	5.8407961
	0	0	-1	4	25.765477	7.90156828

Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
1075.2	0	1.90161096	3.84074161	5.8407961	7.90156828	10

	C_1	C_2	C_3	C_4	b	sol
	4	-1	0	0	3.84074161	1.93314675
	-1	4	-1	0	7.74240705	3.89184541
	0	-1	4	-1	11.7423099	5.89182782
	0	0	-1	4	25.8407961	7.93315598

Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
1209.6	0	1.93314675	3.89184541	5.89182782	7.93315598	10

	C_1	C_2	C_3	C_4	b	sol
	4	-1	0	0	3.89184541	1.95459232
	-1	4	-1	0	7.82497457	3.92652387
	0	-1	4	-1	11.8250014	5.92652859
	0	0	-1	4	25.8918278	7.9545891
Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
1344	0	1.95459232	3.92652387	5.92652859	7.9545891	10

	C_1	C_2	C_3	C_4	b	sol
	4	-1	0	0	3.92652387	1.96915374
	-1	4	-1	0	7.88112091	3.95009107
	0	-1	4	-1	11.881113	5.95008965
	0	0	-1	4	25.9265286	7.96915456
Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
1478.4	0	1.96915374	3.95009107	5.95008965	7.96915456	10

	C_1	C_2	C_3	C_4	b	sol
	4	-1	0	0	3.95009107	1.979047
	-1	4	-1	0	7.91924339	3.96609693
	0	-1	4	-1	11.9192456	5.96609733
	0	0	-1	4	25.9500897	7.97904674
Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
1612.8	0	1.979047	3.96609693	5.96609733	7.97904674	10

	C_1	C_2	C_3	C_4	b	sol
	4	-1	0	0	3.96609693	1.98576682
	-1	4	-1	0	7.94514433	3.97697034
	0	-1	4	-1	11.9451437	5.97697023
	0	0	-1	4	25.9660973	7.98576689
Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
1747.2	0	1.98576682	3.97697034	5.97697023	7.98576689	10

	C_1	C_2	C_3	C_4	b	sol
	4	-1	0	0	3.97697034	1.99033165
	-1	4	-1	0	7.96273704	3.98435624
	0	-1	4	-1	11.9627372	5.98435627
	0	0	-1	4	25.9769702	7.99033163
Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
1881.6	0	1.99033165	3.98435624	5.98435627	7.99033163	10

	C_1	C_2	C_3	C_4	b	sol
	4	-1	0	0	3.98435624	1.99343242
	-1	4	-1	0	7.97468792	3.98937344
	0	-1	4	-1	11.9746879	5.98937343
	0	0	-1	4	25.9843563	7.99343243
Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
2016	0	1.99343242	3.98937344	5.98937343	7.99343243	10

	C_1	C_2	C_3	C_4	b	sol
	4	-1	0	0	3.98937344	1.99553874
	-1	4	-1	0	7.98280586	3.99278153
	0	-1	4	-1	11.9828059	5.99278154
	0	0	-1	4	25.9893734	7.99553874
Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
2150.4	0	1.99553874	3.99278153	5.99278154	7.99553874	10

	C_1	C_2	C_3	C_4	b	sol
	4	-1	0	0	3.99278153	1.99696953
	-1	4	-1	0	7.98832028	3.99509661
	0	-1	4	-1	11.9883203	5.9950966
	0	0	-1	4	25.9927815	7.99696954
Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
2284.8	0	1.99696953	3.99509661	5.9950966	7.99696954	10

Concentraciones en estado estacionario, Diferencias finitas:

$$D\frac{\partial^2 C}{\partial x^2}=0 \Rightarrow \frac{d^2 C}{dx^2}=0$$

Así se llega a:

$$\boxed{C_{i+1}-2C_i+C_{i-1}=0}$$

Se obtiene el siguiente sistema

C_1	C_2	C_3	C_4	b	sol
-2	1	0	0	0	2
1	-2	1	0	0	4
0	1	-2	1	0	6
0	0	1	-2	-10	8

Tiempo (seg)	x=0	x=4	x=8	x=12	x=16	x=20
∞	0	2	4	6	8	10